# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 2nd Semester Supplementary Examination, 2021

## MTMACOR04T-MATHEMATICS (CC4)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* questions from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

   (a) Find the particular integral of the differential equation $(D^2+1)\,y=\cos 2x$.

   (b) Find the Wronskian of the set $\{1-x, 1+x, 1-3x\}$.

   (c) Solve $p^2-3p+2=0,\ p=\dfrac{dy}{dx}$.

   (d) Show that there is a regular singular point of the differential equation

   $$(2x+x^3)\frac{d^2y}{dx^2}-\frac{dy}{dx}-6xy=0.$$

   (e) Show that $\left[\vec{\alpha}+\vec{\beta}\ \vec{\beta}+\vec{\gamma}\ \vec{\gamma}+\vec{\alpha}\right]=2\left[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}\right]$.

   (f) Show that the three vectors $\hat{i}-2\hat{j}+\hat{k},\ 2\hat{i}+\hat{j}-3\hat{k}$ and $-3\hat{i}+\hat{j}+2\hat{k}$ are coplanar.

   (g) If the vectors $\vec{f}=3x\hat{i}+(x+y)\,\hat{j}-ax\hat{k}$ is solenoidal, then find $a$.

   (h) Find the directional derivative of the function $f(x,y,z)=yz+zx+xy$ in the direction of the vector $\vec{u}=\hat{i}-2\hat{j}+\hat{k}$ at the point (1, 2, 0).

2. (a) Solve: $\dfrac{d^2y}{dx^2}+6\dfrac{dy}{dx}+9y=3x^2$. 4+4

   (b) Solve: $\dfrac{d^3y}{dx^3}-\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-y=0$.

3. (a) Solve: $x^3\dfrac{d^3y}{dx^3}-x^2\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}-2y=x^3$. 4+4

   (b) Solve: $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+4y=\cos(\log x)+x\sin(\log x)$.

4. (a) Solve the simultaneous linear equations: $\dfrac{dx}{dt}-7x+y=0,\ \dfrac{dy}{dt}-2x-5y=0$. 4+4

   (b) Solve: $\dfrac{d^2y}{dx^2}-y=x^2\cos x$.

5. (a) Prove that $\left[\vec{\alpha}\times\vec{\beta}\ \vec{\beta}\times\vec{\gamma}\ \vec{\gamma}\times\vec{\alpha}\right]=\left[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}\right]^2$. 4+4

   (b) Show that the four points $2\hat{i}+3\hat{j}-\hat{k},\ \hat{i}-2\hat{j}+3\hat{k}, 3\hat{i}+4\hat{j}-2\hat{k}$ and $\hat{i}-6\hat{j}+6\hat{k}$ are coplanar.

6. (a) Show that the vector $\vec{F}=(2x-yz)\,\hat{i}+(2y+zx)\,\hat{j}+(2z-xy)\,\hat{k}$ is irrotational. 4+4

   (b) Verify Green's theorem in a plane for $\oint\{(x^2+xy)\,dx+xdy\}$ where $C$ is the curve enclosing the region bounded by $y=x^2$ and $y=x$.

7. (a) With the help of vectors prove that the medians of a triangle are concurrent. 4+4

   (b) Prove that the necessary and sufficient condition for a vector $\vec{r}=\vec{f}(t)$ to have a constant direction is $\vec{f}\times\frac{d\vec{f}}{dt}=\vec{0}$.

8. (a) Show that the Wronskian of two solutions of the equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$, $x\in(a,b)$, $P$, $Q$ are functions of $x$, is either identically zero or never zero on ($a$, $b$). 4+4

   (b) Solve: $\sin^2 x\frac{d^2y}{dx^2}=2y$.

9. (a) Solve by using the method of variations of parameters 5+3

$$x^2\frac{d^2y}{dx^2}-2x(1+x)\frac{dy}{dx}+2(1+x)y=x^3$$

   where the integrals in the complementary function are $x$ and $xe^{2x}$.

   (b) Find the integrating factor of the differential equation

$$y(xy+2x^2y^2)\,dx+x(xy-x^2y^2)\,dy=0.$$

**N.B. :** *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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