WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-I Examination, 2020

MATHEMATICS

PAPER-MTMA-I

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *three* questions from the rest

1. Answer any ***four*** questions: $2\times 4 = 8$

(a) Using Descartes' rule of signs find all positive roots, negative roots and complex roots of the following equation:

$$2x^7+3x^6-6x^5-9x^4-8x^3-12x^2+4x+36=0.$$

(b) Find all the values of i^i, where $i^2=-1$.

(c) Find the integers x and y such that $14x+65y=1$.

(d) In the ring $\mathbb{Z}_{12}$ find all the divisors of zero.

(e) If A is a square matrix such that for any non-zero real r, $\det(rA)=r^3\det(A)$, find the order of the matrix A.

(f) Verify whether the set $S=\{(x,\ y):\ 1\le x^2+y^2\le 4\}\subset\mathbb{R}^2$ is a convex set.

(g) Find the point on the circle $r=2a\cos\theta$ which is diametrically opposite to the point $\left(\sqrt{3}a,\ \frac{\pi}{6}\right)$.

(h) Find the point on the x-axis equidistant from the points (1, 2, 7) and (5, 6, –5).

2. (a) Using mathematical induction prove that for all $n\in\mathbb{N}$, $3^{4n+2}+5^{2n+1}$ is divisible by 14. 4

(b) Using Euclid's algorithm find the gcd d of 195 and 1781. Also find the integers x and y such that $d=195x+1781y$. 3+3

(c) Find all the values of z such that $\cos z=2$. 4

3. (a) Find the general solution of $\tan z=2+i$, where $z\in\mathbb{C}$ and $i^2=-1$. 4

(b) If $\alpha,\ \beta,\ \gamma$ are the roots of the equation $x^3-px^2+qx-r=0$, find the value of $\sum\alpha^2\beta^2$. 3

(c) Solve the equation: $8x^3 - 12x^2 - 2x + 3 = 0$, given that the roots are in arithmetic progression. 3

(d) If $f: X \to Y$ is a function, $A \subset X$, prove that $A \subseteq f^{-1}(f(A))$. Show by an example that the inclusion may be proper. 2+2

4. (a) Let ρ be an equivalence relation on a set S. Show that for $a, b \in S$ if $\rho(a) \cap \rho(b) \neq \varnothing$ then $\rho(a) = \rho(b)$, where $\rho(a)$ denotes the equivalence class containing a. 3

(b) X is a set and A, B, C are subsets of X, 2+2

(i) If $A \cap C = B \cap C$ and $A \cap C^c = B \cap C^c$ then prove that $A = B$,

(ii) If $A \cap C = B \cap C$ and $A \cup C = B \cup C$ prove that $A = B$.

(c) In a group $(G, *)$ show that the equations $a * x = b$ and $y * b = c$ have unique solutions in G, where $a, b, c \in G$. 3

(d) Show that a non empty subset H of a group $(G, *)$ is a subgroup of G if and only if for all $a, b \in H$, $a * b^{-1} \in H$. 4

5. (a) If H, K are subgroups of a group G then prove that $H \cap G$ is also a subgroup of G. Show by an example that $H \cup K$ need not be a subgroup of G. 2+2

(b) In an integral domain $(R, +, \cdot)$ prove that $a \cdot b = a \cdot c$ implies $b = c$, where $a, b, c \in R$, $a \neq 0$. Is this result true if R is a ring but not an integral domain? Support your answer. 2+2

(c) Prove without expansion: $\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + dab \\ 1 & d & d^2 & d^3 + abc \end{vmatrix} = 0$, $a, b, c, d \in \mathbb{R}$. 3

(d) For a non-singular $n \times n$ matrix A, prove that $\det(\operatorname{adj} A) = (\det(A))^{n-1}$. 3

6. (a) Write the associated matrix of the quadratic form 1+3

$$f(x, y, z) = 5x^2 + y^2 + 10z^2 - 4yz - 10zx$$

Using congruence operation on the matrix reduce f into normal form.

(b) Find the mid-point of the segment of the line $lx + my + n = 0$ intercepted between the pair of straight lines given by $ax^2 + 2hxy + by^2 = 0$. 3

(c) Using rotation of coordinate axes by an appropriate angle, reduce the equation $3x^2 + 2xy + 3y^2 - 4 = 0$ to standard form. Hence identify the conic represented by it. 3+1

(d) Find the coordinates of the foot of the perpendicular drawn from the point (6, –9, 3) on the plane $2x - 5y + z = 0$. 3

7. (a) Show that the straight lines $\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{-20}$ and $\frac{x+2}{2}=\frac{y+3}{3}=\frac{z+4}{28}$ are intersecting. Find the point of intersection. 3+1

(b) Let $f:A\to B$ and $g:B\to C$ be two functions. Prove that (i) if $g\circ f$ is injective then f is injective and (ii) if $g\circ f$ is surjective then g is surjective. 2+2

(c) Examine whether the solution $(1,\ 0,\ 2)^t$ is a basic solution of the system 2

$$2x-3y+3z=1$$
$$4x+5y+6z=2$$

(d) Solve the following LPP by graphical method: 4

$$\begin{aligned} \text{Maximize} \quad & z=7x+2y \\ \text{Subject to} \quad & -2x+y\le 1 \\ & x+y\le 3 \\ & x\le 2, \\ & x\ge 0,\ y\ge 0. \end{aligned}$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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