WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-I Examination, 2020

MATHEMATICS

PAPER-MTMA-II

Old Syllabus

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 any *three* questions from the rest

1. Answer any ***four*** questions from the following: $2\times 4 = 8$

(a) State Archimedean property of $\mathbb{R}$. Is it true in $\mathbb{Q}$, the set of all rational numbers? 1+1

(b) Prove that every convergent sequence of real numbers is bounded. 2

(c) Verify Bolzano-Weierstrass theorem for the set $\{\frac{n}{n+1} : n\in\mathbb{N}\}$. 2

(d) Prove that $f(x)=\begin{cases} x\sin\frac{1}{x} & , \quad x\neq 0 \\ 0 & , \quad x=0 \end{cases}$ is continuous at $x=0$. 2

(e) Show that $\int_0^1 \frac{x\,dx}{\sqrt{1-x^5}} = \frac{1}{5}\beta(\frac{2}{5},\frac{1}{2})$. 2

(f) Test whether the differential equation 2

$(x^2-4xy-2y^2)\,dx+(y^2-4xy-2x^2)\,dy=0$ is exact or not.

(g) Using Beta-Gamma function evaluate $\int_0^{\pi/2}\sin^6 x.\cos^8 x\,dx$. 2

(h) Find the unit vector perpendicular to both the vectors $2\hat{i}-6\hat{j}-3\hat{k}$ and $4\hat{i}+3\hat{j}-\hat{k}$. 2

2. (a) Let A and B be two non-empty bounded sets of real numbers, $a=\sup A$, $b=\sup B$ and $C=\{x+y : x\in A,\ y\in B\}$. Show that $\sup C=a+b$. 3

(b) If $\{x_n\}$ be a convergent sequence and converges to l, then show that the sequence $\{|x_n|\}$ converges to $|l|$. Is the converse true? Justify your answer. 3+1+1

(c) Examine whether the sequence $\{\frac{n-1}{n+1}\}$ is a Cauchy sequence or not. 2

(d) Prove that the sequence 3

$\sqrt{2}$, $\sqrt{2\sqrt{2}}$, $\sqrt{2\sqrt{2\sqrt{2}}}$, …… converges to 2.

(e) State Sandwich theorem on sequence. 1

3\. (a) Show that the set $\{x \in \mathbb{R} \mid 0 < x < 1\}$ is non-denumerable. 3

(b) Define the terms (i) interior point of a set S (ii) open set in $\mathbb{R}$. 1+1

(c) Prove that intersection of two open sets in $\mathbb{R}$ is again an open set in $\mathbb{R}$. 2

(d) Find the derived set of $S = \left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\}$. 3

(e) If f is continuous at $x = c$ and $f(c) \neq 0$ then show that there is a nbd. of c, where $f(x)$ and $f(c)$ keep the same sign. 3

(f) What do you mean by piecewise continuous function? 1

4\. (a) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then prove that $n(I_{n+1} + I_{n-1}) = 1$. 3

(b) Evaluate : $\int_0^{\pi/2} \frac{dx}{5 + 4\cos x}$ 3

(c) Show that $2^{2m-1}\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \sqrt{\pi}\,\Gamma(2m),\ m > 0$. 4

(d) If ρ_1 and ρ_2 are radii of curvatures at two extremities of any chord of the cardioid $r = a(1 + \cos\theta)$ passing through the pole, prove that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$. 4

5\. (a) Find the asymptotes of the curve $x^3 + x^2 y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$. 5

(b) Find the envelopes of the line $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by $a^2 + b^2 = c^2$ [c is a constant]. 4

(c) Find the general solution and singular solution of the differential equation $y = px + \sqrt{a^2 p^2 + b^2}$, $\left[p = \frac{dy}{dx}\right]$. 2+3

6\. (a) Solve: $(D^2 + 4)y = \sin 2x \left[D^2 = \frac{d^2}{dx^2}\right]$. 4

(b) Solve by the method of undetermined coefficients: 5

$$(D^2 - 3D + 2)y = 14\sin 2x - 18\cos 2x$$

(c) Solve: $\frac{d^2 y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = e^x \sec x$ by reducing it to normal form. 5

7. (a) Show that $[\vec{\alpha}+\vec{\beta},\ \vec{\beta}+\vec{\gamma},\ \vec{\gamma}+\vec{\alpha}] = 2[\vec{\alpha},\ \vec{\beta},\ \vec{\gamma}]$. 3

(b) Find a unit vector in the plane of the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-2\hat{k}$ which is perpendicular to the vector $2\hat{i}+\hat{j}-\hat{k}$. 3

(c) A force $\vec{P} = 4\hat{i}-3\hat{k}$ passes through a point A whose position vector is $2\hat{i}-2\hat{j}+5\hat{k}$. Find the moment of $\vec{P}$ about the point B with position vector $\hat{i}-3\hat{j}+\hat{k}$. 3

(d) Determine a constant a, so that the vector function $\vec{f} = (x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal. 2

(e) Prove that $\nabla\cdot(\phi\vec{A}) = (\nabla\phi)\cdot\vec{A}+\phi(\nabla\cdot\vec{A})$. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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